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A GENERAL CHARACTERIZATION OF INTERIM EFFICIENT MECHANISMS FOR INDEPENDENT LINEAR ENVIRONMENTS

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Abstract

We consider the class of Bayesian environments with independent types, and utility functions which are both quasi-linear in a private good and linear in a one-dimensional private-value type parameter. We call these *independent linear environments*. For these environments, we fully characterize interim efficient allocation rules which satisfy interim incentive compatibility and interim individual rationality constraints. We also prove that they correspond to decision rules based on virtual surplus maximization, together with the appropriate incentive taxes. We demonstrate how these techniques can be applied easily to the design of auctions, markets, bargaining rules, public good provision, and assignment problems.

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Key words: public goods, mechanism design, interim efficiency, incentive compatibility, private values

1 Introduction

This paper presents a general approach for solving optimal mechanism design problems in a the class of Bayesian environments with independent types, and utility functions which are both quasi-linear in a private good and linear in a one-dimensional independent private value type parameter. We call these *independent linear environments.* We fully characterize interim efficient allocation rules which also satisfy interim incentive compatibility and interim individual rationality constraints in these environments. We also prove that these solutions correspond to decision rules based on a virtual cost-benefit criterion, together with the appropriate incentive taxes. We demonstrate through a series of illustrations how these techniques can be applied easily to auction theory, bargaining, public goods, and other standard problems of mechanism design. These solutions, some of which are by now well-known are derived as simple corollaries of our characterization theorem.

Many papers have now been written on the optimal mechanism design for Bayesian environments.[1] While a variety of technical approaches have been taken, most of these papers share a common mathematical structure, but this common structure is not transparent, as these techniques are scattered across a number of articles, each of which focuses on a specific application or feature of the general problem. Here, we exploit that common structure to give a full characterization of interim efficient allocation rules for what we call *linear independent environments.* These environments have quasi-linear utility, additivity in taxes in the feasibility constraints, and linearity of utilities in a one dimensional independent private-value type. The general model embodies both public good problems and private

[1]See, for example, d'Aspremont and Gérard-Varet (1979), Dudek, Kim, and Ledyard (1995), Laffont and Maskin (1979, 1982), Myerson and Satterthwaite (1983), Myerson (1981), Gresik (1996), Cornelli (1996), Wilson (1985), Wilson (1985, 1993), Mailath and Postlewaite (1990), Ledyard and Palfrey (1994, 1999a, 1999b, 2002), Coughlan (1999), Cramton, Gibbons, and Klemperer (1987), Cramton and Palfrey (1990), Crémer, d'Aspremont, and Gérard-Varet (1999), and others.

good problems in a single framework.

As is standard, we use the revelation principle, to enable us characterize efficient allocation rules by restricting attention to direct revelation mechanisms. We use the separation result of d'Aspremont and Gérard-Varet (1979) which allows the separate computation of feasible incentive taxes. We will use an insight of Myerson and Satterthwaite (1983) which reduces individual rationality constraints to a single constraint that does not involve the incentive taxes. Our approach is closest to the original Mirrlees (1971) analysis of optimal taxation for income redistribution, and Wilson's (1993) later study of ex ante optimal trading procedures.

In contrast to the above papers, this paper is concerned with interim efficient allocation rules, using a concept first introduced by Holmstrom and Myerson (1983). An allocation rule is interim efficient if there exists no other allocation rule that makes no type of any agent worse off and makes some types of some agents better off. It is the natural generalization of Pareto optimality to Bayesian environments where agents have private information. There are only a handful of paper that explore the properties of interim efficient allocation rules, and these are limited to a few applications.[2]

The next section presents the basic notation and the model. Section 3 presents the characterization results and proofs. Section 4 shows how the characterization is simplified in the regular case and section 5 illustrates this approach with several applications to both public and private goods environments. We make some concluding remarks in section 6.

[2]See Gresik (1996) and Wilson (1985) for applications to bilateral trade, particularly double auctions. See Coughlan (2000), Laussel and Palfrey (2002) and Ledyard and Palfrey (1994,1999a, 1999b, 2002) for applications to public good mechanisms. Perez-Nievas (2000) investigates the interim efficiency of Groves mechanisms.

2 The Model

There are N individual agents. An *outcome* consists of a social allocation and a profile of taxes. A *social allocation* is an M-vector, denoted $x = (x^1, \ldots, x^M)$ which is an element of a feasible set $X \subseteq R^M$ for some $M > 0$. Furthermore, the *cost* of the social allocation is given by $C(x)$, and $a = (a^1, ..., a^N) \in R^N$ is a *profile of taxes* for the agents, which must collectively be sufficient to cover the cost of x. We denote the set of feasible profiles of taxes, given an allocation x, by $\mathbf{A}(x) = \{a \in R^N \mid \sum_{i=1}^N a^i \geq C(x)\}$. Formally, a *feasible outcome* is a pair $(x, a) \in Z$ where Z is the subset of $X \times R^M$ such that $a \in \mathbf{A}(x)$ for all $x \in X$.

Each player has a type, t^i. We assume that each individual knows his own type and does not know the types of the other individuals. We assume that the types are *independently distributed*, with the (common knowledge) cdf of i's type denoted $F_i(\cdot)$ and the support of F_i is $T^i = [\underline{t}^i, \overline{t}^i] \subseteq R$. We assume F_i has a continuous positive density on T_i. Note that $\underline{t}^i < 0$ is allowed. The von Neumann Morgenstern utility function for type t^i of agent i for an allocation (x, a) is assumed to take the form $\mathbf{V}(x, a, t^i) = t^i q^i(x) - a^i$.[3]

An *allocation rule* is a mapping from $T = T^1 \times \cdots \times T^N$ into Z. A *mechanism* is a game form consisting of a message set for each agent and an outcome function that maps message profiles into probability distributions over the set of feasible allocations. A *direct mechanism* is a mechansim in which the message set for each agent is simply T^i. By the revelation principle, any allocation rule that results from equilibrium in any mechanism is also an equilibrium allocation rule of an incentive compatible, direct mechanism. Therefore, the rest of the paper only considers direct mechanisms.

A strategy for i in a direct mechanism is a mapping $\sigma^i : T^i \rightarrow T^i$: that is, a decision rule that specifies a reported type for each possible type. We refer to the identity mapping

[3] In many applications, $q^i(x)$ is the quantity consumed by agent i in the social allocation x. However, this is just one of several possible interpretations of q.

as the *truthful strategy*, and denote it by $\Im$, so $\Im(t^i) = t^i$. We denote a feasible direct mechanism simply as a function, $\eta : T \to Z$.[4] We denote the social allocation component of η at type profile t by $x(t)$ and the tax profile by $a(t)$. We will refer to the pair $(q^i(x), a^i)$ as i's allocation under η.

2.1 Incentive Compatibility

Besides resource feasibility, the two restrictions on η considered in this paper are incentive compatibility and individual rationality. Incentive compatibility requires that it is a Bayesian equilibrium of η for all agents to adopt a strategy of truthfully reporting their type. Given a strategy profile $\sigma^i : T^i \to T^i$ and mechanism, η, let the interim utility of type t^i of agent i, assuming all others truthfully report their type, be denoted by:

$$\widehat{U}^i(\eta, t^i, \sigma^i) = \int_T [t^i q^i[x(\sigma^i(t^i), t^{-i})] - a^i(\sigma^i(t^i), t^{-i})] dF(t|t^i)$$

For convenience, we use a simplified notation for the case when $\sigma^i = \Im$, denoting $U^i(\eta, t^i) \equiv \widehat{U}^i(\eta, t^i, \Im)$.

Definition 1 *A direct mechanism η is (interim)* incentive compatible *if and only if $U^i(\eta, t^i) \geq \widehat{U}^i(\eta, t^i, \sigma^i)$ for all i, t^i, σ^i.*

2.2 Individual Rationality

We are also interested in allocation rules η which satisfy an interim individual rationality constraint. This means each type of each agent will be at least as well off, at the interim

[4]By the linearity of these environments, there is no loss in restricting attention to deterministic mechanisms.

stage, by participating, as they would be by not participating. We assume the interim expected utility of not participating in the mechanism does not depend on the mechanism, but can depend on type. We denote this non-participation value by $U^{0i}(t^i)$.

Definition 2 *A direct mechanism η satisfies (interim)* individual rationality *if and only if $U^i(\eta, t^i) \geq U^{0i}(t^i)$ for all i, t^i.*

2.3 Interim Efficiency

Definition 3 *A direct mechanism η is* interim efficient *iff (a) η is feasible, (b) η is (interim) incentive compatible and (c) η satisfies the (interim) individual rationality constraints for each i and $\nexists\ \widehat{\eta}$ such that (a) $\widehat{\eta}$ is feasible, (b) $\widehat{\eta}$ is (interim) incentive compatible and (c) $\widehat{\eta}$ satisfies the (interim) individual rationality constraints for each i and $U^i(\widehat{\eta}, t^i) \geq U^i(\eta, t^i)$ for all i, t^i, and $U^i(\widehat{\eta}, t^i) > U^i(\eta, t^i)$ for some i and for all $t^i \in \widetilde{T}^i \subset T^i$, where $\widetilde{T}^i$ has strictly positive measure relative to T^i.*

The following well-known result[5] is stated below, without proof:

Lemma 1 *A direct mechanism η is an interim efficient mechanism iff $\exists\ \lambda = \left\{\lambda^i : T^i \rightarrow R_+\right\}_{i=1}^n$ with $\int_{\underline{t}^i}^{\bar{t}i} \lambda^i(t^i) dF^i(t^i) > 0$ for some i, such that η solves maximize $\sum_{i=1}^N \int_{\underline{t}^i}^{\bar{t}i} \lambda^i(t^i) U^i(\eta(t), t^i) dF^i(t^i)$ subject to (a) η is feasible, (b) η is (interim) incentive compatible and (c) η satisfies the (interim) individual rationality constraint.*

We now proceed to characterize that set of interim efficient mechanisms.

[5] See Holmstrom and Myerson (1983).

3 The Characterization

3.1 Incentive Compatibility

3.1.1 Characterization

We first identify incentive compatible mechanisms in a useful way. For (interim) smooth mechanisms[6] when preferences are linear, the characterization of incentive compatibility in terms of derivatives is well-known. There are basically two features of such mechanisms. First, an envelope condition is satisfied, namely that the total derivative of the interim utility for i with respect to type when players adopt truthful strategies is equal to the partial derivative with respect to type (i.e., fixing the reports of all agents). Second, the interim utility to i under truthful reporting is convex in i's type. This is stated formally below, without proof.

Lemma 2 *(Rochet, 1987): If $\widehat{U}^i$ is linear in t^i, and U^i is continuously differentiable, then η is incentive compatible if and only if*

$$\nabla_{t^i} U^i(\eta, t^i) = \nabla_{t^i} \widehat{U}^i(\eta, t^i, \sigma^i)$$

$$U^i(\eta, t^i) \text{ is convex in } t^i.$$

3.1.2 Reduced form allocations

For our problem, this characterization of incentive compatibility can be explained in terms of each type's reduced form allocation; that is, the expected value of that type's allocation

[6] By interim smooth we mean that the reduced form allocation rules $Q^i(t^i) = \int q^i(x(t))dF_{-i}(t_{-i})$ and $A^i(t^i) = \int a^i(t)dF_{-i}(t_{-i})$ are twice differentiable in t^i.

under the mechanism, when all agents report truthfully. The reduced form social allocation of type t^i is denoted is $Q^i(t^i) \equiv \int_T q^i[x(t)]dF(t|t^i)$, and type t^i's expected tax is denoted by $A^i(t^i) \equiv \int_T a^i(t)dF(t|t^i)$. Therefore, it follows directly from above that $\nabla_{t^i} U^i(\eta, t^i) = Q^i(t^i)$, and $\nabla_{t^i} \widehat{U}^i(\eta, t^i, \Im) = Q^i(t^i) + t^i \frac{dQ^i(t^i)}{dt^i} - \frac{dA^i(t^i)}{dt^i}$ where A^i_t and Q^i_t are the derivatives of A^i and Q^i, respectively. Finally, U^i is convex in t^i if and only if $\frac{dQ^i(t^i)}{dt^i} \geq 0 \ \forall t^i \in T^i$. Thus η is incentive compatible iff, for $t^i \in t$,

$$t^i \frac{dQ^i(t^i)}{dt^i} - \frac{dA^i(t^i)}{dt^i} = 0 \tag{IC1}$$

and

$$Q^i_{t^i}(t^i) \geq 0. \tag{IC2}$$

3.2 The Constrained Optimization Problem

Collecting the above results, we can state the following theorem.

Theorem 1 *An allocation rule $\eta = (x^*, a^*)$ is interim efficient among the set of feasible incentive compatible mechanisms satisfying individual rationality, if and only if there exists a system of type-dependent welfare weights, $\{\lambda^i : T \rightarrow R^+\}_{i=1}^N$, with $\sum_{i=1}^N \int_{T^i} \lambda^i(t^i)dF^i(t^i) > 0$*

for some i, such that (x, a) solves the following constrained maximization problem:

$$\max_{x(\cdot),a(\cdot)} \int_T \left\{ \sum_{i=1}^{N} \lambda^i(t^i)[t^i q^i(x(t)) - a^i(t)] \right\} dF(t)$$

$$\begin{aligned}
\textit{subject to} &: \\
x(t) &\in X \ \forall t \\
\sum_{i=1}^{N} a^i(t) &\geq C(x(t)) \ \forall t \\
A^i(t^i) &= A^i(\underline{t}^i) + \int_{\underline{t}^i}^{t^i} s dQ^i(s) \ \forall i \textit{ and } t^i \\
\frac{dQ^i(t^i)}{dt^i} &\geq 0 \ \forall i \textit{ and } t^i \\
t^i Q^i(t^i) - A^i(t^i) &\geq U^{0i}(t^i) \ \forall i \textit{ and } t^i.
\end{aligned}$$

These constraints are, respectively, ex post feasibility for x, ex post feasibility for (a, x), incentive compatibility ($IC1$ and $IC2$), and interim individual rationality. In Ledyard and Palfrey (1999) we adopted the approach of Mirrlees (1971) and Wilson (1993) to characterize the solution to this program in the special case of pure public goods, ignoring the last set of constraints corresponding to individual rationality.. Here we use a more general framework to highlight commonalties across the literature.

We first use a separation result of d'Aspremont and Gérard-Varet (1979) to establish feasibility of simple transfer schemes that can be constructed in a balanced way to provide the correct incentive scheme.

3.3 Incentive Taxes

To make the notation a bit simpler below, we provide a few new definitions here. We first define an agent's *minimum net utility* from a mechanism which is the surplus received by the worst-off type of agent i, assuming incentive compatible transfers.

Definition 4 *Given Q^i and U^{0i}, let*

$$\begin{aligned}
L^i(Q^i, U^{0i}) &\equiv \min_{t^i}[t^i Q^i(t^i) - \int_{\underline{t}^i}^{t^i} s dQ^i(s) - U^{0i}(t^i)] \\
&= \min_{t^i}[\underline{t}^i Q^i(\underline{t}^i) + \int_{\underline{t}^i}^{t^i} Q^i(s) ds - U^{0i}(t^i)] \\
&= \underline{t}^i Q^i(\underline{t}^i) + \min_{t^i}[\int_{\underline{t}^i}^{t^i} Q^i(s) ds - U^{0i}(t^i)]
\end{aligned}$$

Remark 1 *$L^i(Q^i, U^{0i}) - A^i(\underline{t}^i)$ is i's **minimum net utility** given incentive compatible taxation. A feasible incentive compatible mechanism satisfies interim individual rationality if and only if $L^i(Q^i, U^{0i}) - A^i(\underline{t}^i) \geq 0$ for all i.*

Next we define the expected budget surplus of an incentive compatible allocation rule (summed over all agents).

Definition 5 *Given an allocation rule x let*

$$G(x) \equiv \sum_{i=1}^{N} \left[\int_{\underline{t}^i}^{\bar{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) - \underline{t}^i Q^i(\underline{t}^i) \right] - \int_T C(x(t)) dF(t).$$

Remark 2 *Notice that $G(x) + \sum_{i=1}^{N} A^i(\underline{t}^i) = \left[\sum_{i=1}^{N} \int_T a^i(t) dF(t) \right] - \int_T C(x(t)) dF(t)$. It is the **ex-ante budget surplus**, given incentive compatible taxation.*

The next step involves the separation of the transfer problem (the choice of a) from the allocation problem (the choice of x), which is a well-known result.[7] We include a proof for completeness.

[7] See, for example, d'Aspremont and Gérard-Varet (1979).

Lemma 3 *If $x : T \to X$ satisfies $dQ^i/dt^i \geq 0$, and $\{A^{0i}\}_{i=1}^N$ is any collection of N constants, then $\exists$ a such that (x, a) is incentive compatible and feasible and $A^{0i} = A^i(\underline{t}^i)$ for all i, if and only if*

$$G(x(\cdot)) + \sum_{i=1}^{N} A^i(\underline{t}^i) \geq 0.$$

Proof. For each i and t, let

$$\begin{aligned} a^i(t) &= \alpha^{oi} + \int_{\underline{t}^i}^{t^i} sdQ^i(s) - \frac{1}{N-1}\sum_{j\neq i}\int_{\underline{t^j}}^{t^j} sdQ^j(s) + \\ &\quad \frac{1}{N}[C(x(t)) - C^i(t^i) + \frac{1}{N-1}\sum_{j\neq i} C^j(^j)] \end{aligned}$$

where $C^i(t^i) = \int_T C(x(t))dF(t|t^i)$ and

$$\alpha^{0i} = A^{0i} + \frac{1}{N-1}\sum_{j\neq i}\int_{\underline{t^j}}^{\overline{t}^j}\int_{\underline{t^j}}^{t^j} sdQ^j(s)dF^j(t^j) - \frac{1}{N}\int_T C(x(t))dF(t)$$

If $a^i(t)$ is computed this way then for each t, then

$$\sum_{i=1}^{N} a^i(t) = \sum_{i=1}^{N} \alpha^{oi} + C(x(t))$$

Therefore, (x, a) is feasible if and only if $\sum_i \alpha^{oi} \geq 0$, or, equivalently,

$$\begin{aligned}
\sum_{i=1}^{N}\left\{A^{0i}+\frac{1}{N-1}\sum_{j\neq i}\int_{\underline{t}^j}^{\bar{t}^j}\int_{\underline{t}^j}^{t^j} sdQ^j(s)dF^j(t^j)-\frac{1}{N}\int_T C(x(t))dF(t)\right\} &\geq 0 \\
&\Leftrightarrow \\
\sum_{i=1}^{N}A^{0i}+\sum_{i=1}^{N}\int_{\underline{t}^i}^{\bar{t}i}\int_{\underline{t}^i}^{t^i} sdQ^i(s)dF^i(t^i)-\int_T C(x(t))dF(t) &\geq 0 \\
&\Leftrightarrow \\
\sum_{i=1}^{N}A^{0i}+G(x(\cdot)) &\geq 0.
\end{aligned}$$

To verify that (x, a) is incentive compatible, observe first that $dQ^i/dt^i \geq 0$ by hypothesis and

$$\begin{aligned}
A^i(t^i) &= \alpha^{0i}+\int_{\underline{t}^i}^{t^i} sdQ^i(s) \\
&\quad -\frac{1}{N-1}\sum_{j\neq i}\int_{\underline{t}^j}^{t^j} sdQ^j(s)dF^j(t^j) \\
&\quad +\frac{1}{N}\int_T C(x(t))dF(t) \\
&= A^{0i}+\int_{\underline{t}^i}^{t^i} sdQ^i(s).
\end{aligned}$$

so both $IC1$ and $IC2$ are satisfied and $A^{0i} = A^i(\underline{t}^i)$ for all i. ■

The inequality in the statement of the lemma requires that given incentive compatible taxation, ex ante expected taxes are greater than or equal to ex ante expected costs. In other words, it is only the ex ante budget balance constraint that is binding. Since one can always find A^{0i} such that (11) holds, one can always find type-dependent transfers to balance the budget.

3.4 Combining Incentive Constraints and Voluntary Participation

The next step is to obtain a more convenient form of the individual rationality constraint. This is done by combining it with the feasibility and incentive constraints and then summing over agents.

Lemma 4 *If $x : T \to X$ satisfies $dQ^i/dt^i \geq 0$, then there exists a such that (x, a) is incentive compatible, feasible, and satisfies individual rationality if and only if*

$$G(x(\cdot)) + \sum_{i=1}^{N} L^i(Q^i(\cdot), U^{0i}) \geq 0.$$

Proof. (only if) Let a be such that (x, a) is incentive compatible, feasible, and satisfies individual rationality. Incentive compatibility implies that there exist $\{A^{0i}\}_{i=1}^{N}$ such that:

$$A^i(t^i) = A^{0i} + \int_{\underline{t}^i}^{t^i} sdQ^i(s) \; \forall i, t^i.$$

The individual rationality constraint is

$$t^i Q^i(t^i) - A^i(t^i) - U^{0i}(t^i) \geq 0 \; \forall i, t^i.$$

Combining the two gives:

$$t^i Q^i(t^i) - \int_{\underline{t}^i}^{t^i} sdQ^i(s) - A^{0i} - U^{0i}(t^i) \geq 0 \; \forall i, t^i,$$

or, equivalently,

$$\min \left\{ t^i Q^i(t^i) - \int_{\underline{t}^i}^{t^i} sdQ^i(s) - U^{0i}(t^i) \right\} \geq A^{0i} \; \forall i$$

or

$$L^i(Q^i(\cdot), U^{0i}) \geq A^{0i} \; \forall i$$

Summing over i gives:

$$\sum_{i=1}^{N} L^i(Q^i(\cdot), U^{0i}) \geq \sum_{i=1}^{N} A^{0i}$$

From Lemma 3, $\sum_{i=1}^{N} A^{0i} \geq -G(x(\cdot))$, so

$$G(x(\cdot)) + L^i(Q^i(\cdot), U^{0i}) \geq 0$$

(if) Suppose $G(x(\cdot)) + \sum_{i=1}^{N} L^i(Q^i(\cdot), U^{0i}) \geq 0$ and. For each i, let $A^{0i} = L^i(Q^i(\cdot), U^{0i})$. Summing over i gives:

$$\sum_{i=1}^{N} L^i(Q^i(\cdot), U^{0i}) = \sum_{i=1}^{N} A^{0i}$$

which implies

$$G(x(\cdot)) + \sum_{i=1}^{N} A^{0i} \geq 0.$$

From Lemma 3, this implies the existence of a such that (x, a) is feasible and incentive compatible for all i and $A^{0i} = A^i(\underline{t}^i)$ for all i. By construction $A^{0i} = L^i(Q^i(\cdot), U^{0i})$ which implies that (x, a) satisfies individual rationality. ■

Remark 3 *A useful corollary is: There exist* $\{A^{0i}\}_{i=1}^{N}$ *such that* $G(x(\cdot)) + \sum_i A^{0i} \geq 0$ *and* $L^i(Q^i(\cdot), U^{0i}) \geq A^{0i}$ *for all* i *if and only if* $G(x(\cdot)) + \sum_i L^i(Q^i(\cdot), U^{0i}) \geq 0$.

Remark 4 *Lemma* 4 *is useful in checking whether a particular incentive compatible* (x, a) *satisfies IR. It does not help in computing an appropriate* x^* *to solve the optimization problem.*

Remark 5 *As an example, in Myerson and Satterthwaite (1983) there are two types of agents, a buyer, B, and a seller, S, and* $C(x) = 0$.

Individual rationality reduces to:

$$t_0^i Q^i(t_0^i) - L^i(Q^i(\cdot), U^{0i}) = 0 \text{ for } B$$

and:

$$t_0^i Q^i(t_0^i) - L^i(Q^i(\cdot), U^{0i}) = -\int_{\underline{t}^i}^{\overline{t}^i} Q^i(s)ds \text{ for } S,$$

since each trader can guarantee himself the no trade outcome. B has no endowment, but S has the option to keep the object and receive $U^{0i}(t^i) = t^i$*. Using these two identities, and the constraint,* $G(x(\cdot)) + \sum_i L^i(Q^i(\cdot), U^{0i}) \geq 0$*, yields the familiar inequality (extended for arbitrary numbers of buyers and sellers), :*

$$\sum_{i \in B} \int_{\underline{t}^i}^{\overline{t}^i} (t^i - \frac{1 - F^i(t^i)}{f^i(t^i)})Q^i(t^i)dF^i(t^i) + \sum_{j \in S} \int_{\underline{t}^j}^{\overline{t}^j} (t^j + \frac{F^j(t^j)}{f^j(t^j)})Q^j(t^j)dF^j(t^j) \geq 0$$

3.5 Characterization of Interim Efficient Allocations

We introduce one more piece of notation and a simple lemma to ease notation.

Definition 6 *If* $\lambda^{0i} = \int_{\underline{t}^i}^{\overline{t}^i} \lambda^i(t^i)dF^i(t^i) > 0$*, let* $\Lambda^i(t^i) = \frac{1}{\lambda^{0i}} \int_{\underline{t}^i}^{t^i} \lambda^i(s)dF^i(s)$.[8] *If* $\lambda^{0i} = 0$*, then* $\Lambda^i(t^i) = 0$.

Lemma 5

$$\int_{\underline{t}^i}^{\overline{t}^i} \lambda^i(t^i)[t^i Q^i(t^i) - \int_{\underline{t}^i}^{t^i} sdQ^i(s)]dF^i(t^i) = \lambda^{0i} \left[\underline{t}^i Q^i(\underline{t}^i) + \int_{\underline{t}^i}^{\overline{t}^i} \left(\frac{1-\Lambda^i(t^i)}{f^i(t^i)}\right) Q^i(t^i)dF^i(t^i)\right].$$

[8] *Wilson (1985) refers to* $\Lambda^i(\cdot)$ *as the conditional welfare weights of agent i.*

Proof. Integrate by parts. ■

We can use lemmas 3,4, and 5 to provide a slimmer version of the problem in theorem 1 where we characterize interim efficiency.

Theorem 2 *Given $x : T \to X$, there exists a such that $\eta = (x, a)$ is interim efficient iff there exist non-negative type-dependent welfare weights, $\{\lambda^i\}_{i=1}^N$, where $\sum_i \lambda^{0i} > 0$, and N constants, $\{A^{0i}\}_{i=1}^N$, such that $(x, \{A^{0i}\}_{i=1}^N)$ solves,*

$$
\begin{aligned}
& \max_{x\in X} \sum_{i=1}^{N} \lambda^{0i} \left[\int_{\underline{t}^i}^{\bar{t}^i} \left(\frac{1-\Lambda^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) + \underline{t}^i Q^i(\underline{t}^i) - A^{0i} \right] \\
& \textit{subject to} \\
0 \leq \;& L^i(Q^i(\cdot), U^{0i}) - A^{0i} \textit{ for all } i \\
0 \leq \;& G(x(\cdot)) + \sum_{i=1}^{N} A^{0i} \\
& 0 \leq dQ^i(t^i)/dt^i \textit{ for all } i,\ t^i
\end{aligned} \tag{1}
$$

Proof. Follows from lemmas 3,4, and 5. ■

Without individual rationality, this problem simplifies. First, note that the (ex ante) welfare weights must all be equal. That is, without loss of generality, $\lambda^{0i} = 1$ for all i. Otherwise, the problem has no solution since one can always improve welfare by arbitrarily large transfers between agents with different ex ante weights. Second, the constant transfers, $\{A^{0i}\}_{i=1}^N$, have no welfare consequences beyond their sum. The following corollary summarizes this.

Corollary 1 *Given $x : T \to X$, there exists a such that $\eta = (x, a)$ is interim efficient (without individual rationality), iff there exist non-negative type-dependent welfare weights,*

$\{\lambda^i\}_{i=1}^N$, *such that for all i,j,* $\lambda^{0i} = \lambda^{0j} > 0$ *and* x *solves:*

$$\max_{x \in X} \sum_{i=1}^{N} \left[\int_{\underline{t}^i}^{\bar{t}^i} \left(\frac{1 - \Lambda^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) + \underline{t}^i Q^i(\underline{t}^i) + G(x(\cdot)) \right]$$

subject to

$$0 \leq dQ^i(t^i)/dt^i \text{ for all } i,\ t^i$$

Proof. Obvious. ■

4 The regular case

In this section, we characterize the solution to the problem in theorem 2, in the case where constraint 1 is not binding, and identify conditions under which the solution to this relaxed problem satisfies the missing constraint. When this is true, we refer to the problem as *the regular case.* We adopt a Kuhn-Tucher approach to solving for an optimum.

4.1 Kuhn-Tucker Conditions

From the Kuhn-Tucker Theorem and Theorem 2 we know that in the regular case, (x^*, a^*) is interim efficient if and only if there exists a non-negative system of type-dependent welfare weights, $\{\lambda^i\}_{i=1}^N$, with $\sum_{i=1}^N \lambda^{0i} > 0$ for some i, individual multipliers, $\{\rho^i\}_{i=1}^N$, a multiplier, δ, and A^{*0}, such that (x^*, A^{*0}) solves

$$\max_{x \in X, A^0} \sum_{i=1}^{N} \lambda_o^i \left[\int_{\underline{t}^i}^{\bar{t}^i} \left(\frac{1 - \Lambda^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) + \underline{t}^i Q^i(\underline{t}^i) - A^{0i} \right] \tag{2}$$

$$+ \sum_{i=1}^{N} \rho^i \left[L^i(Q^i(\cdot), U^{0i}) - A^{0i} \right] + \delta \left[G(x(\cdot)) + \sum_{i=1}^{N} A^{0i} \right]$$

and

$$
\begin{aligned}
\rho^i &\geq 0 \text{ for all } i \\
0 &\leq L^i(Q^i(\cdot), U^{0i}) - A^{0i} \text{ for all } i \\
0 &= \rho^i \left[L^i(Q^i(\cdot), U^{0i}) - A^{0i}\right] \text{ for all } i \\
\delta &\geq 0 \\
0 &\leq G(x(\cdot)) + \sum_{i=1}^{N} A^{0i} \\
0 &= \delta[G(x(\cdot)) + \sum_{i=1}^{N} A^{0i}]
\end{aligned}
$$

4.2 Solving for $\mathbf{A}_0^i$

Suppose $[\lambda, \rho, \delta, x^*, A^{*0}]$ solves (21). First, observe that, at $[x^*, A^{*0}]$, the first order conditions of 2 with respect to A^{0i} are necessary for an optimum, and this implies:

$$-\lambda^{0i} - \rho^i + \delta = 0 \text{ for all } i.$$

Define $\overline{\lambda} \equiv \max_i\{\lambda^{0i}\}$. Then $\rho^i \geq 0$ implies $\delta \geq \overline{\lambda} \geq \lambda^{0i}$ for all i. Since $\sum_{i=1}^{N} \lambda^{0i} > 0$, this immediately implies $\delta > 0$ and $G(x^*(\cdot)) + \sum_{i=1}^{N} A^{*0i} = 0$. So as long as x satisfies $G(x^*(\cdot)) + \sum_{i=1}^{N} L^i(Q^{*i}(\cdot), U^{0i}) \geq 0$, we can solve for A^{0i} (and hence a as well). This is summarized in the following theorem.

Theorem 3 $\delta \geq \overline{\lambda} > 0$ *and* $G(x^*(\cdot)) + \sum_{i=1}^{N} L^i(Q^{*i}(\cdot), U^{0i}) \geq 0$ *if and only if there exist*

$\{A^{*0i}\}_{i=1}^N$ and $\{\rho^i\}_{i=1}^N$ solving the Kuhn-Tucker conditions above. Furthermore,

$$\begin{aligned}
\rho^i &= \delta - \lambda^{0i} \text{ for all } i \\
\sum_{i=1}^N A^{*0i} &= -G(x^*(\cdot)) \\
\lambda^{0i} &< \overline{\lambda} \Longrightarrow A^{*0i} = L^i(Q^{*i}(\cdot), U^{0i}) \text{ for all } i
\end{aligned}$$

Proof. Follows immediately from the definition of $\overline{\lambda}$ and the assumption that $\sum_i \lambda^{0i} > 0$. ■

Remark 6 *$A^{0i} = L^i(Q^{*i}(\cdot), U^{0i})$ implies that all the agents with low ex ante welfare weights are taxed up to the limit of their IR constraint.*

Remark 7 *$\sum_{i=1}^N A^{*0i} = G(x^*(\cdot))$ implies that $\sum_{i=1}^N a^i(t) = C(x^*(t))$ for all t. Hence there is no inefficiency in production (the budget always balances).*

Remark 8 *If $\lambda^{0i} = \overline{\lambda}$, A^{*0i} is unconstrained and determined as residual profit from the other agents for whom $A^{*0i} = L^i(Q^{*i}(\cdot), U^{0i})$. But if $G(x^*(\cdot)) + \sum_{i=1}^N L^i(Q^{*i}(\cdot), U^{0i}) = 0$, then $A^{*0i} = L^i(Q^{*i}(\cdot), U^{0i})$ for all i.*

4.3 Solving for x

Having dispensed with A_0^i and ρ^i, using (22) we can restate the Kuhn-Tucker conditions of Theorem XX as $\exists\{\lambda^i\}_{i=1}^N$ and with $\sum_{i=1}^N \lambda^{0i} > 0$ such that:

$$\begin{aligned}
x^* &\in \arg\max_{x\in F} \sum_{i=1}^N \lambda^{0i}\left[\int_{\underline{t}^i}^{\overline{t}^i}\left(\frac{1-\Lambda^i(t^i)}{f^i(t^i)}\right)Q^i dF^i + \underline{t}^i Q^i(\underline{t}^i)\right]. \\
&\quad +\delta G(x(\cdot)) + \sum_{i=1}^N (\delta-\lambda^{0i})L^i(Q^i(\cdot),U^{0i}) \\
0 &\leq \delta - \overline{\lambda} \\
0 &\leq G(x(\cdot)) + \sum_{i=1}^N L^i(Q^i(\cdot),U^{0i}) \\
0 &= [\delta-\overline{\lambda}]\left(G(x(\cdot)) + \sum_{i=1}^N L^i(Q^i(\cdot),U^{0i})\right)
\end{aligned}$$

Since $\delta \geq \overline{\lambda} > 0$, this can be rewritten as $\exists\{\lambda^i\}_{i=1}^N$ and $\delta > 0$ with $\sum_{i=1}^N \lambda^{0i} > 0$ such that:

$$\begin{aligned}
x^* &\in \arg\max_{x\in X} \sum_{i=1}^N \frac{\lambda^{0i}}{\delta}\left[\int_{\underline{t}^i}^{\overline{t}^i}\left(\frac{1-\Lambda^i(t^i)}{f^i(t^i)}\right)Q^i(t^i)dF^i(t^i) - M^i(Q^i(\cdot),U^{0i})\right] \\
&\quad +\sum_{i=1}^N\left[\int\left(t^i - \frac{1-F^i(t^i)}{f^i(t^i)}\right)Q^i(t^i)dF^i(t^i) + M^i(Q^i(\cdot),U^{0i})\right] - \int_T C(x(t))dF(t) \\
&\quad 0 \leq G(x(\cdot)) + \sum_{i=1}^N L^i(Q^i(\cdot),U^{0i}) \\
0 &\leq \delta - \overline{\lambda} \\
0 &= (\delta-\overline{\lambda})\left(G(x(\cdot)) + \sum_{i=1}^N L^i(Q^i(\cdot),U^{0i})\right)
\end{aligned}$$

where $M^i(Q^i(\cdot),U^{0i}) = L^i(Q^i(\cdot),U^{0i}) - \underline{t}^i Q^i(\underline{t}^i) = \min_{t^i}\left[\int_{\underline{t}^i}^{t^i} Q^i(s)ds - U^{0i}(t^i)\right]$. This implies the following theorem.

Theorem 4 *Suppose $[x^*, A^{0*}]$ solves 2. Then $\exists$ a^* such that (x^*, a^*) is interim efficient if and only if there exist $\{\lambda^i\}_{i=1}^N$ and $\delta > 0$ with $\sum_{i=1}^N \lambda^{0i} > 0$ such that:*

$$
\begin{aligned}
& x^* \in \arg\max_{x \in X} \sum_{i=1}^N \int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} + \frac{\lambda^{0i}}{\delta} \frac{1 - \Lambda^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \\
& + \sum_{i=1}^N (1 - \frac{\lambda^{0i}}{\delta}) M^i(Q^i(\cdot), U^{0i}) - \int C(x(t)) dF(t) \\
& 0 \leq \sum_{i=1}^N \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) + M^i(Q^i(\cdot), U^{0i}) \right] \\
& - \int_T C(x(t)) dF(t) \\
0 \quad \leq \quad & \delta - \overline{\lambda} \\
0 \quad = \quad & (\delta - \overline{\lambda}) \{ \sum_{i=1}^N \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) + M^i(Q^i(\cdot), U^{0i}) \right] \\
& - \int_T C(x(t)) dF \}
\end{aligned}
$$

Remark 9 *If $(1 - \frac{\lambda^{0i}}{\delta}) M^i(Q^i(\cdot), U^{0i}) = 0$, the term of the maximand in large square brackets:*

$$
W^i(t^i, \lambda^i, \delta) \equiv t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} + \frac{\lambda^{0i}}{\delta} \frac{1 - \Lambda^i(t^i)}{f^i(t^i)}
$$

is the virtual valuation of type t^i (Myerson (1981), Wilson (1985), Gresik (1996), and Ledyard and Palfrey (1999a, 1999b)). The virtual valuation is equal to the player's private value type, t^i, with adjustments due to two factors. The first adjustment is for information rents, the $-\frac{1-F^i(t^i)}{f^i(t^i)}$ term. The second adjustment is due to possible distortions arising from redistribution of income, which occurs because of the welfare weights, and is captured in the expression, $\frac{\lambda^{0i}}{\delta} \frac{1-\Lambda^i(t^i)}{f^i(t^i)}$.

Remark 10 *If* $(1-\frac{\lambda^{0i}}{\delta})M^i(Q^i(\cdot),U^{0i}) \neq 0$, *in many cases it simply reduces to a straightforward adjustment of the expression for the agent's virtual valuation. For example, in Myerson and Satterthwaite's (1983) study of ex ante efficient bargaining mechanisms,* $\Lambda^i(t^i) = F^i(t^i)$ *and* $\lambda^{0i} = 1$ *for both the buyer and the seller, but there are individual rationality constraints. For the buyer,* $M^i(Q^i(\cdot),U^{0i}) = 0$, *and for the seller* $M^i(Q^i(\cdot),U^{0i}) = \int_{\underline{t}^i}^{\overline{t}^i} Q^i(t^i)dt^i$. *Therefore* $W^i = t^i - \frac{\delta-1}{\delta}\frac{1-F^i(t^i)}{f^i(t^i)}$ *for the buyer and* $W^i = t^i + \frac{\delta-1}{\delta}\frac{F^i(t^i)}{f^i(t^i)}$ *for the seller, for suitable choice of* δ. *Thus it seems the "virtual valuation" interpretation of the solution is valid quite generally, requiring only minor adjustment when* $(1-\frac{\lambda^{0i}}{\delta})M^i(Q^i(\cdot),U^{0i}) \neq 0$ *for some* i.

For the rest of this section, we assume $(1-\frac{\lambda^{0i}}{\delta})M^i(Q^i(\cdot),U^{0i}) = 0$.

Remark 11 *In regular independent linear environments, interim efficient mechanisms can be derived by simply modifying the original first best problem by replacing the valuation* t^i, *with the virtual valuation* $W^i(t^i,\lambda^i,\delta)$. *This leads to a natural algorithm for the relaxed problem, which involves solving an ex post problem, using virtual valuations in the place of the actual private valuations.* ***Step 1****: Set* $\delta = \overline{\lambda}$, *and for each* t *let* $x^*_\delta(t)$ *solve* $\max_{x\in X}\sum_{i=1}^N W^i(t^i,\lambda^i,\delta)q^i(x) - C(x)$. *If* $\sum_{i=1}^N \int_{\underline{t}^i}^{\overline{t}^i}\left(t^i - \frac{1-F^i(t^i)}{f^i(t^i)}\right) q^i\left(x^*_\delta(t)\right) dF(t\mid t^i) - \int_T C\left(x^*_\delta(t)\right) dF(t) \geq 0$, *this is the solution, and go to step 4. If not, then* ***Step 2****: For every* $\delta > \overline{\lambda}$, *for each* t *let* $x^*_\delta(t)$ *solve* $\max_{x\in X}\sum_{i=1}^N W^i(t^i,\lambda^i,\delta)q^i(x) - C(x)$. ***Step 3****: Find the minimum value of* δ^* *such that* $\sigma(\delta^*) = 0$, *where*

$$\sigma(\delta) = \sum_{i=1}^N \int_{\underline{t}^i}^{\overline{t}^i}\left(t^i - \frac{1-F^i(t^i)}{f^i(t^i)}\right) q^i\left(x^*_\delta(t)\right) dF(t\mid t^i) - \int_T C\left(x^*_\delta(t)\right) dF(t).$$

and calculate $a^*(t)$ *using the formula in the proof of 3.* ***Step 4****: The solution is* $x^*_\delta(t)$.

Remark 12 *The relaxed problem simply drops the second order conditions,* $Q^i_{t^i}(t^i) \geq 0\ \forall i, t^i$, *so the question is: When is the solution to the "relaxed" problem also a solution to the original* *problem? A complete answer to this question will give a full characterization of the regular case. A partial answer is easier to find. Specifically, a sufficient condition for* $Q^i_{t^i}(t^i) \geq 0$ $\forall i, t^i$ *is that* $\frac{\partial W^i}{\partial t^i} \geq 0$, *for all* t^i, i. *That is virtual valuations are monotone in type. As Gresik (1996) and Ledyard and Palfrey (1999a, 1999b) point out, this boils down to a joint condition on priors* F_i and *welfare weights* λ. *The standard condition (i.e. without welfare weights or participation constraints), that* $t^i - \frac{1-F^i(t^i)}{f^i(t^i)}$ *be increasing in* t^i *for all* i, *is neither necessary nor sufficient. For example if* F^i *is uniform on* $[0,1]$ *then* $W^i(t^i) = t^i - \frac{1-F^i(t^i)}{f^i(t^i)} + \frac{\lambda^{0i}}{\delta}\left(\frac{1-\Lambda^i(t^i)}{f^i(t^i)}\right)$ $= t^i - [1-F^i(t^i)] + \frac{\lambda^{0i}}{\delta}[1-\Lambda^i(t^i)] = 2t^i - 1 - \frac{1}{\delta}[1 - \int_{\underline{t}^i}^{t^i} \lambda^i(s)dF^i(s)]$. *So* $\frac{\partial W^i}{\partial t^i} = 2 - \frac{1}{\delta}\lambda^i(t^i)$. *For the special case of constant welfare weights, say* $\lambda = 1$, *this implies* $\frac{\partial W^i}{\partial t^i} = 2 - \frac{1}{\delta} > 0$ *since* $\delta > 1$, *so the solution to the relaxed problem for the uniform case is always optimal.*[9] *But for interim efficiency, which allows for nonconstant* $\lambda(t^i)$, *one generally needs further restrictions in order to satifsy the second order conditions of the full optimization problem. For example, in the uniform case described above, the solution to the relaxed problem satisfies the second order conditions of the full problem if and only if* $\lambda^i(t^i) \leq 2\delta$ *for all* i, t^i.

Remark 13 *When* $\frac{\partial W^i}{\partial t^i}(\widehat{t^i}) < 0$ *for some* i, $\widehat{t^i}$, *the constrained optimal solution can be obtained by a procedure known as"ironing" (Guesnerie and Laffont 1985, Rochet and Choné 2000); that is,* Q^i *must be constant over some interval, which results in flat regions, sometimes referred to as bunching of types. This raises a question of which interim efficient mechanisms are missed by the algorithm based on virtual valuations.*

Remark 14 *Myerson and others refer to a regular case occurring when*

$$\frac{\partial}{\partial t^i}\left[t^i - \frac{1-F^i(t^i)}{f^i(t^i)}\right] \geq 0 \text{ for all } i.$$

[9] *The case of constant welfare weights corresponds ex ante efficiency.*

This continues to be the appropriate condition if $\lambda(t^i)$ is constant in t^i for all i. If $\lambda^i(t^i)$ is not constant in t^i for i the regularity condition requires the additional distributional term, $\frac{\lambda^{0i}}{\delta}\frac{1-\Lambda^i(t^i)}{f^i(t^i)}$. That is:

$$\frac{\partial}{\partial t^i}\left[t^i - \frac{1-F^i(t^i)}{f^i(t^i)} + \frac{\lambda^{0i}}{\delta}\left(\frac{1-\Lambda^i(t^i)}{f^i(t^i)}\right)\right] \geq 0$$

The derivative of of the additional term can be positive or negative, depending on how the "welfare weights" behave. Hence, nonconstant type-contingent welfare weights can lead to either more or less bunching, compared to the ex ante solution. An illustration of the difference can be seen when F^i is the uniform districtution, where $W^i(t^i, \lambda^i, \delta) = t^i - \delta(1 - t^i)(1-\delta)(t^i - \Lambda^i)$ and $\frac{\partial W^i}{\partial t^i} = 2 - \frac{1}{\delta}\lambda^i(t^i)$. Notice that we are in the Myerson "regular" case since $\frac{d\left[t^i - \frac{1-F^i(t^i)}{f^i(t^i)}\right]}{\partial t^i} = 2 > 0$. But, unless $\lambda \leq 2\delta$, there will be t^i such that $\frac{\partial W^i}{\partial t^i} < 0$ so the "normal case" will not apply, and bunching will result. Observe that since the condition is $\lambda \leq 2\delta$, regularity is satisfied for a wider range of welfare weights for higher values of δ, which correspond (loosely speaking) to more binding participation constraints.

Remark 15 *Without IR, simply let $\delta = \lambda^{0i}$ for all i. Then, for the regular case, the solution is any x^* such that*

$$x^* \in \arg\max_{x\in X} \sum_{i=1}^{N} \int_{\underline{t}^i}^{\overline{t}^i} \left(t^i + \frac{F^i(t^i) - \Lambda^i(t^i)}{f^i(t^i)}\right) Q^i(t^i)dF^i(t^i) - \int_T C(x(t))dF(t)$$

Remark 16 *Without IR, if $\lambda^i(t^i) = 1$ for all i and t^i, then $F^i(t^i) = \Lambda^i(t^i)$ for all i and t^i, implying:*

$$x^* \in \arg\max_{x\in X} \int_T \left[\left(\sum_{i=1}^{N} t^i q^i(x(t)\right) - C(x(t))\right] dF(t)$$

or first best.

Remark 17 *For ex ante neutral welfare weights, $\lambda_o^i = \overline{\lambda}$ for all i. In this case, the IR constraint is non-binding iff $\delta = \overline{\lambda}$, so,*

$$x^* \in \arg\max_{x \in X} \sum_{i=1}^{N} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i + \frac{F^i(t^i) - \Lambda^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right] - \int_T C(x(t)) dF(t)$$

implies

$$0 \leq \sum_{i=1}^{N} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) - M^i(Q^i(\cdot), U^{0i}) \right] - \int_T C(x(t)) dF(t),$$

and hence x^* is interim efficient.

There are several special cases which illustrate these results.

Example 1 *Consider the d'Aspremont and Gérard-Varet (1979) case where $\lambda^i(t^i) = 1$ for all i and t^i because of ex-ante efficiency. Then $\Lambda^i(t^i) = F^i(t^i)$, and there are no individual rationality constraints, so $\delta = \lambda^{0i} = 1$. Thus, $W^i = t^i$ and the first best solution is achieved.*

Example 2 *Consider the Ledyard andPalfrey (1999a,1999b) case where there are ex ante neutral welfare weights, so $\lambda^{i0} = 1$, but no individual rationality so also $\delta = 1$. Then $W^i = t^i - \frac{\Lambda^i(t^i) - F^i(t^i)}{f^i(t^i)}$.*

5 Applications

We turn to applications of the characterization of interim efficient mechanisms in several different economic environments. Summarizing the previous section, a specific application consists of a specification of:

$$N, X, C(x), \{T^i, F^i, q^i : X \to \Re, U^{0i} : T^i \to \Re\}_{i=1}^N$$

To find a specific interim efficient allocation for such an environment, one specifies a collection of type-contingent welfare weights, $\{\lambda^i : T^i \to \Re^+\}_{i=1}^N$ and applies the techniques outlined in the previous section.

Most of the applications we consider here have been studied in separate papers, so part of the point of this section is to illustrate how all of these models are contained as special cases of the general framework in this paper. Remember that, by *interim efficienct,* we refer to mechanisms that are efficient for some type-dependent weighting scheme, relative to the class of feasible, incentive compatible direct mechanisms satisfying interim individual rationality. By *independent linear* environments, we refer to Bayesian settings with independent private values, with utility functions linear in a one-dimensional type, and a transferable commodity, and with feasibility also additive in that commodity. We will assume throughout this section that we are in the "regular case" where

$$W^i(t^i, \lambda^i, \delta) \equiv t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} + \frac{\lambda^{0i}}{\delta} \frac{1 - \Lambda^i(t^i)}{f^i(t^i)}$$

if $(1 - \frac{\lambda^{0i}}{\delta})M^i(Q^i(\cdot), U^{0i}) = 0$ and $\frac{\partial W^i(t^i,\alpha)}{\partial t^i} \geq 0$ for all t^i. Where $(1 - \frac{\lambda^{0i}}{\delta})M^i(Q^i(\cdot), U^{0i}) \neq 0$ for some i we make the appropriate adjustment to that agent's virtual valuation. We first consider public goods environments, and then private goods environments.

5.1 Public Goods

5.1.1 Pure Public Goods

Without IR constraints In d'Aspremont and Gérard-Varet (1979), they characterized ex-ante efficient mechanisms for a pure (nonexcludable) public good environment with independent linear types, without participation constraints. That paper also discovered the balancing incentive taxes described in proof of lemma 3. In that environment, the incentive constraints are not binding. That is, the first best solution is possible. Ledyard and Palfrey (1999a, 1999b) consider the set of all interim efficienct mechanisms, without participation constraints, and show that the classical first best solution is interim efficient only for the special case of constant welfare weights. The intuition is that non constant welfare weights imply that there should be redistribution of the private good across types. The balancing incentive taxes one needs to make first best allocations incentive efficient will generally distort the redistibution unless welfare weights are constant.

In the notation of this paper, the pure public goods model in Ledyard and Palfrey (1999b) is $X = [0, 1]$, $C(x) = Kx$, $q^i(x) = x$, individual rationality was not required (i.e., $\delta = 1$). For the regular case, given the welfare weights, $\lambda^i : T^i \longrightarrow \Re^+$, an interim efficient mechanism satisfies:

$$x^*(t) \in \arg\max_x \left(\sum_i W^i(t^i, \lambda^i) - K \right) x.$$

where $W^i(t^i, \lambda^i) = t^i + \frac{F^i(t^i) - \Lambda^i(t^i)}{f^i(t^i)}$. Provided the second order conditions are satisfied, the efficient public decision always involves a simple cost benefit calculation: produce $x = 1$ if and only if the sum of the virtual valuations exceeds the cost of production. Otherwise, produce $x = 0$. For ex ante efficient mechanisms, $\lambda^i(t^i) = 1$ for all i and t^i, and simple calculations verify $W^i(t^i) = t^i$, so the problem is regular for all F. Hence ex ante efficient public decisions correspond to the classical first best.

If λ is not constant, then interim efficient mechanisms will generally have over-production or under-production relative to the first best levels. Furthermore, this also holds for variety of specifications of $q^i(x)$ and $C(x)$. Suppose for example that $q^i(t^i) = q(t^i)$ for all i, q is concave, increasing, and C is convex and increasing. Then a first-best decision, x^o, satisfies $\sum_i t^i \frac{\partial q(x^o)}{\partial x} = \frac{\partial C(x^o)}{\partial x}$. For interim efficient mechanisms, given a set of welfare weights, a necessary condition for interim efficiency in the regular case is:

$$\sum_{i=1}^{N} W^i(t^i, \lambda^i) \frac{\partial q(x^*)}{\partial x} = \frac{\partial C(x^*)}{\partial x}.$$

Therefore, if $W^i(t^i, \lambda^i) > t^i$ for all t then $x^*_\lambda(t) \geq x^o(t)$ and over-production is interim efficient. Indeed, $W^i(t^i, \lambda^i) > t^i$ occurs, for example, if $\lambda^i(t^i)$ is increasing in t^i. That is, when higher types are more heavily weighted than lower types, over-production is a more efficient way to relax incentive compatibility constraints than transfers, a^i. The economic intuition behind this result is the following. First, since higher types are weighted more heavily, welfare is increased either by shifting taxes from high types to low types or by producing the public good more often. However, the only way to shift the tax burden from higher types to lower types, without violating incentive compatibility or feasibility, is to produce the public good less often, which would make high types worse off. This intuition is not dependent on the linearity of q^i in x or the linearity of the production technology.

With IR constraints Finally, we look at the case of interim efficiency with individual rationality constraints, which is handled by this general framework. Two easy facts can be observed for regular environments. For simplicity, we deal only with the case of constant welfare weights (ex ante efficiency), but the same results hold with general welfare weights.

For this case,

$$W^i(t^i, \lambda^i, \delta) \equiv t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} + \frac{\lambda^{0i}}{\delta}\frac{1 - \Lambda^i(t^i)}{f^i(t^i)}$$

does not reduce to $W^i(t^i, \lambda^i, \delta) \equiv t^i$, because $\delta > 1$. Instead, one gets

$$W^i(t^i, \lambda^i, \delta) \equiv t^i - \left(1 - \frac{\lambda^{0i}}{\delta}\right)\frac{1 - F^i(t^i)}{f^i(t^i)}$$

Therefore virtual valuations are lower, so the efficient choice of x is always lower with individual rationality constraints than without. For example, suppose $X = [0, 1], C(x) = Kx$ and $q^i(x) = x$. Then for some realizations of t such that $\sum_i t^i - K$ is positive, but not very large, it will be necessary to produce zero because there is not enough surplus to cover incentive costs without violating individual rationality.

The second observation follows from Mailath and Postlewaite (1990), and addresses ex ante efficiency in large N environments. To illustrate their result in the context of our model, suppose costs increase linearly in N so that $C(x) = Nc(x)$. Further, assume $q^i(x) = q(x)$ for all i, q is concave and c is convex. Ex ante efficient mechanisms solve, for each t, $\frac{1}{N}\sum_i W^i(t^i, \delta)\frac{\partial q(x^*(t))}{\partial x} = \frac{\partial c(x^*(t))}{\partial x}$ for suitably chosen δ. As $N \to \infty$, by the law of large numbers, for any fixed value of δ, $\frac{1}{N}\left(\sum_i W^i(t^i, \delta)\right)$ converges to the expected virtual valuation, call it, $\overline{W}(\delta)$. So, for any given $\delta > 1$, as $N \to \infty, x^*(t) \to \overline{x}(\delta)$ for all t, where $\overline{x}(\delta)$ is defined by $\overline{W}(\delta)\frac{\partial q(\overline{x}(\delta))}{\partial x} = \frac{\partial c(\overline{x}(\delta))}{\partial x}$. Thus $Q^i(t^i) \to q(\overline{x}(\delta))$ for all i and t^i. Incentive compatibility then requires that $A^i(t^i) \to \overline{A}$, a constant, and feasibility requires that $\overline{A} = c(\overline{x}(\delta))$, the average cost share. Finally, given $Q^i(t^i) = q(\overline{x}(\delta))$ and $\overline{A}^i = c(\overline{x}(\delta))$, the individual rationality constraint is that $t^i q(\overline{x}(\delta)) \geq c(\overline{x}(\delta))$ for all t^i. But then $\sum_i t^i q(\overline{x}(\delta)) \geq Nc(\overline{x}(\delta))$ for all t. If $\underline{t}^i < 0$ and $c(x) > 0$ for all $x > 0$, then we must have $q(\overline{x}(\delta)) = 0$. So, unless it is first-best optimal to produce positive amounts of the public good for all realizations of t, the *individually rational* ex ante efficient level of public good production goes to zero as

$N \to \infty$.

5.1.2 Excludable Public Goods

Without IR constraints An excludable public good is one for which i's consumption of the good is allowed to be any y^i such that $0 \leq y^i \leq x$. So $U^i = t^i q^i(y^i) - a^i$, $x \in R_+$. Here, $(x, y^i, \ldots, y^N)$ is feasible if and only if $x \geq 0, 0 \leq y^i \leq x$ for $i = 1, \ldots, N$.

The social decision for an interim efficient mechanism thus solves

$$\begin{aligned} &\max_{(x, y^i, \ldots, y^N)} \sum_{i=1}^{N} W^i(t^i, \lambda^i) q^i(y^i) - h(x) \\ \text{subject to } x &\in R_+, 0 \leq y^i \leq x. \end{aligned}$$

So assuming $\frac{dq^i}{dy^i} > 0$, and second order conditions are satisfied, interim efficient allocations satisfy, for each t,

$$x^*(t) \in \arg\max_x \sum_{i=1}^{N} \max \left\{ W^i(t^i, \lambda^i), 0 \right\} q^i(x) - h(x)$$

and $y^i = x$ iff $W^i(t^i, \lambda^i) \geq 0$.

For *ex-ante efficiency*, $W^i(t^i) = t^i$. So in these cases:

$$x^* \in \arg\max_x \sum_{i=1}^{N} \max \left\{ t^i, 0 \right\} q^i(x) - h(x)$$

and $y^i = x$ iff $t^i \geq 0$. If $\underline{t}^i \geq 0$, then $y^i = x$ always and there is no difference between the ex ante efficient mechanisms in pure public good case and the excludable case. The threat of exclusion provides no help in relaxing incentive constraints, simply because incentive constraints are not binding to begin with.

For *interim efficiency*, $W^i = t^i - \frac{\Lambda^i(t^i) - F^i(t^i)}{f^i(t^i)}$. So $y^i = x$ iff $t^i \geq \frac{\Lambda^i(t^i) - F^i(t^i)}{f^i(t^i)}$ and $x^*(t) \in \arg\max_x \sum_i \max\left\{t^i - \frac{\Lambda^i(t^i) - F^i(t^i)}{f^i(t^i)}, 0\right\} q^i(x) - h(x)$. It follows that if the welfare weights favor low types then $\Lambda^i(t^i) - F^i(t^i) > 0$ and there is lower production of x and more types are excluded than under the ex ante efficient mechanism. If the weights favor high types then $\Lambda^i(t^i) - F^i(t^i) < 0$ and there is higher production and less exclusion realtive to the ex ante efficient mechanism.[10]

With IR constraints Cornelli (2000) examined ex ante efficiency with individual rationality constraints for excludable public goods. Applying our techniques, in regular environments, the ex ante optimal mechanism we get, for suitably chosen $\delta > 1$:

$$\begin{aligned} &\max_{x \geq 0} \sum_{i=1}^{N} \max\left\{t^i - \frac{\delta - 1}{\delta} \frac{1 - F(t^i)}{f^i(t^i)}, 0\right\} q^i(x) - h(x), \\ \text{and } y^i &= x \text{ iff } t^i \geq \frac{\delta - 1}{\delta} \frac{1 - F^i(t^i)}{f^i(t^i)}, \end{aligned}$$

where δ is minimized on $\delta \geq 1$ subject to

$$\sum_{i=1}^{N} \int_{T^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)}\right) q^i(y^i_\delta) dF^i(t^i) \geq \int_T h(x_\delta(t)) dF(t).$$

Several observations can be made. If individual rationality is binding then it may be ex-ante or interim-efficient to exclude some t_i even though ex post it would be efficient to include them. This occurs when, for some i there exist $t^i > 0 > t^i - \frac{\delta-1}{\delta}\frac{1-F^i(t^i)}{f^i(t^i)}$. So exclusion does provide help in relaxing the individual rationality constraints. However, note that the individual rationality constraint is not always binding. If $\underline{t}^i f(\underline{t}^i) \geq 1$, then $W^i(t^i, \delta) \geq 0$, so

[10] Coughlan (1999) studies excludable public goods with congestion costs, and no IR constraint. The results are similar, with an adjustment term for the crowding externality.

exclusion is never used.[11]

Limiting results with exclusion for $N \to \infty$ Does exclusion provide a way around the Malaith and Postlewaite (1990) result? Let us look at ex ante efficiency for the linear symmetric case where $q^i(y^i) = y^i$, $C(x) = Nkx$ where $k = K/N$, $F^i = F^j$ for all i, j, and $x \in [0,1]$. The ex ante efficient mechanism (ignoring IR constraints) has the property that as $N \to \infty$, $x \to 1$ if $E[\max\{t^i, 0\}] > k$ and $x \to 0$ if $E_{t^i}[\max\{t^i, 0\}] < k$.

Next consider the ex ante efficient solution when the IR constraint constraint is binding. For the appropriate $\delta > 1$, let t^0_δ solve $W^i(t^0_\delta, \delta) = 0$, or $t^0_\delta - \frac{\delta-1}{\delta}\frac{1-F^i(t^0_\delta)}{f^i(t^0_\delta)} = 0$. That is, t^0_δ is the boundary type separating those who are excluded from those who are not excluded, given δ . Then the individual rationality constraint reduces to $\int_{t^0_\delta}^{\bar{t}} \left(t - \frac{1-F(t)}{f(t)}\right) dF(t) \geq k$ if $x \to 1$. So if there is a value of $\delta > 1$ such that $t^0_\delta(1 - F(t^0_\delta)) \geq k$ then positive production of the public good occurs even as $N \to \infty$, and some types will be excluded.

Does interim efficiency change these properties as $N \to \infty$? Without individual rationality constraints, $W^i = t^i - \frac{\Lambda^i(t^i)-F^i(t^i)}{f^i(t^i)}$ and i is excluded iff $W^i(t^i, \lambda^i) < 0$. Let t^{oi} be the solution to $W^i(t^{oi}, \lambda^i) = 0$ and consider the regular case where $\frac{\partial W^i(t^i,\lambda^i)}{\partial t^i} \geq 0$. Now $x \to 1$ as $N \to \infty$ iff $E[\max\{W^i, 0\}] \geq k$. That is $x \to 1$ as $N \to \infty$ iff $t^{oi}(\Lambda^i\,(t^{oi}) - F^i(t^{oi})) + \int_{Tt^o}(s\lambda(s)dF(t^i) \geq k$. So there will be positive production of the public good. Also if low types are favored, (that is, λ is decreasing in type), then relative to ex ante efficiency there will be more exclusion and less production. The opposite is true if high types are favored.

Next consider the limiting solution with individual rationality constraints. For suitable δ, an interim efficient mechanism excludes all $t \leq \widehat{t}$ where $\widehat{t}$ satisfies

$$\widehat{t} - \frac{1 - F(\widehat{t})}{f\left(\widehat{t}\right)} + \frac{1}{\delta}\frac{1 - \Lambda(\widehat{t})}{f\left(\widehat{t}\right)} = 0$$

[11] For the uniform distribution $\underline{t}f(\underline{t}) \geq 1$ iff $\underline{t} \geq (1/2)\bar{t}$.

In the limit, either $q(t) = 1$ for all t or $q(t) = 0$ for all t depending on whether $\int_{\hat{t}}^{\bar{t}} W^i(t, \lambda, \delta) dF^i(t) \gtreqless k$. To determine the suitable δ, note that the participation constraint is satisfied in the limit if

$$\sum_{i=1}^{N} \int_{T^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i_\delta(t^i) dF^i(t^i) \geq \int_T C(x_\delta(t)) dF(t)$$

iff

$$\hat{t}(1 - F(\hat{t})) \geq k$$

Therefore, choose t^o such that $t^o(1 - F(t^o)) = k$, if it exists, and choose δ to solve $t^o - \frac{1-F(t^o)}{f(t^o)} + \frac{1}{\delta}\frac{1-\Lambda(t^o)}{f(t^o)} = 0$. Then in the limit, an interim efficient mechanism excludes i if $t^i < t^o$ and produces if and only if $\int_{t^o}^{\bar{t}} W^i(t, \lambda, \delta) dF^i(t) \geq k$ which holds if and only if $\int_{t^o}^{\bar{t}} \lambda(t) dt \geq k$.

When F is uniform on $[0, 1]$, $W^i(t, \lambda, \delta) = 2t - 1 + \frac{1}{\delta}(1 - \Lambda(t))$ and $(1 - t^o) = k$. If $k > \frac{1}{4}$ then $x = 0$ and $N \to \infty$. If $k \leq \frac{1}{4}$, then $x \to 1$ as $N \to \infty$, $t^o = \frac{1}{2} - \frac{1}{2}\sqrt{1 - 4k}$ and $\frac{1-\delta}{\delta} = \frac{2t^o - \Lambda(t^o)}{1 - \Lambda(t^o)}$. $W^i(t, \lambda, \delta) = 2t - 1 + \left(\frac{\sqrt{1-4k}}{1-\Lambda(t^o)}\right)(1 - \Lambda(t^o))$. And $\frac{dW^i}{dt}(t^o) = 2 - \left(\frac{\sqrt{1-4k}}{1-\Lambda(t^o)}\right)\lambda(t^o)$. Notice that, as is true in general, t^0 does not depend on the welfare weights. When the individual rationality constraints are binding as $N \to \infty$, the cut-off point for exclusion, t^o, is such that if all who are not excluded pay equal shares then costs are exactly covered - no matter which type is preferred. So in the limit all interim mechanisms converge to the ex ante optimal mechanism. If tthere is a t^o such that $t^o(1 - F(t^o)) = k$, then $x = 1$, t is excluded iff $t \leq t^o$, those excluded pay nothing and those not excluded pay t^o. If there is no such t^o then $x = 0$.

5.2 Private Goods

Myerson (1981), Myerson and Satterthwaite (1983), Wilson (1993), Cramton, Gibbons, and Klemperer (1988) and others have studied *ex ante* efficient mechanisms for linear private good environments.[12] In our notation, for all of these settings,

$$\begin{aligned} X &= \{x \in \Re^N \mid \sum_{i-1}^{N} x^i \leq J\} \\ U^i &= t^i q^i - a^i \\ C(x) &= 0 \end{aligned}$$

where J is the quantity of private good available.

In the exchange environments considered here, the set of agents is divided into two categories, buyers and sellers. Buyers are assumed to have no endowment of the good to be exchanged, but unlimited amounts of the transferable utility good. The buyers and sellers have $q^i(x) = x^i$. Each seller owns one unit of the good to be exchanged and this is reflected in their individual rationality constraints, which are type dependent. For example $U^{0i}(t^i) = t^i$ if i is a supplier of 1 unit. These problems neatly divide themselves into specific applications, depending on the number of buyers and sellers. We distinguish the following four applications in this way:

1. *Bargaining*: 1 buyer and 1 seller

2. *Markets*: $I > 1$ buyers and $J > 1$ sellers

3. *Auctions*: I buyers and 1 seller (or 1 buyer and J sellers)

4. *Assignment*: I buyers and 0 sellers

[12] Gresik (1996) and Wilson (1985) consider interim efficient mechanisms in private good settings.

5.2.1 *Bargaining*: One buyer and one seller

In the simplest case, due to Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983) $||B|| = ||S|| = 1$. the original Myerson-Satterthwaite problem. Let $p(t) = 1 - x^s(t)$ be the probability of a trade, and write the mechanism design problem as first choosing, for each possible $\delta \geq 1$, a probability $p_\delta(t) \in [0,1]$, such that, for each $t = (t^b, t^s)$, $p_\delta(t)$ maximizes:

$$p_\delta(t)\left\{\left(t^b - \frac{1-F^b(t^b)}{f^b(t^b)} + \frac{1}{\delta}\frac{1-\Lambda^b(t^b)}{f^b(t^b)}\right) - \left(t^s + \frac{F^s(t^s)}{f^s(t^s)} - \frac{1}{\delta}\frac{\Lambda^s(t^s)}{f^s(t^s)}\right)\right\}$$

and then select the minimum value of δ that satisfies:

$$\int_T \left[\left(t^b - \frac{1-F^b(t^b)}{f^b(t^b)}\right) - \left(t^s + \frac{F^s(t^s)}{f^s(t^s)}\right)\right] p_\delta(t^b, t^s) dF^b(t^b) dF^s(t^s) \geq 0$$

Denoting that minimum value by δ^*, the resulting mechanism, p_{δ^*}, is interim efficient.

With uniform priors on $[0,1]$, trade occurs iff

$$t^b - t^s \geq \frac{1}{2}\left[\frac{\delta-1}{\delta} + \frac{1}{\delta}\left\{\int_0^{t^b} \lambda^B(s)ds - \int_0^{t^s} \lambda^s(s)ds\right\}\right].$$

In the ex ante case $\lambda^b(t) = 1 = \lambda^s(t)$, so trade occurs iff $(2 - \frac{1}{\delta})(t^b - t^s) \geq \frac{1}{\delta}$ where δ just satisfies the individual rationality constraint. To contrast this with other possible interim efficient mechanisms, consider two possible alternative welfare weights, $\lambda_1(t) = 2t$ and $\lambda_2(t) = 2(1-t)$. λ_1 weights high types more heavily than low types, λ_2 does the opposite. Note that $\Lambda^1 = t^2$ and $\Lambda^2 = 2t - t^2$. Letting $\Lambda^b = \Lambda^1$ and $\Lambda^s = \Lambda^2$ we can see that trade will occur iff $(2 - \frac{1}{\delta})(t^b - t^s) - \frac{1}{\delta} - \frac{\delta-1}{\delta}[t^b(t^b-1) + t^s(t^s-1)] \geq 0$. Since $[t^b(t^b-1) + t^s(t^s-1)] < 0$, this inequality will be satisfied for more (t^b, t^s) than was the case for ex ante efficieny for the same α. Thus more trade occurs.[13] Reversing the Λ's so that $\Lambda^b = \Lambda^2$ *and* $\Lambda^s = \Lambda^1$ leads to less trade than is ex ante efficient.

5.2.2 *Markets*: Many buyers and many sellers

In this framework, the results of Myerson and Satterthwaite (1983) and Gresik (1996) are easily extended to n buyers and m sellers. Let $X = \{x | \sum_{i=1}^{n+m} x^i = m\}$. So, $\exists a^*$ such that (x^*, a^*) is incentive efficient if and only if $\exists \lambda, \delta$ such that

$$x^* \in \arg\max_{x \in X} \sum_{i \in B} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} + \frac{\lambda^{0i}}{\delta} \frac{1 - \Lambda^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right] \tag{3}$$

$$+ \sum_{i \in S} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i + \frac{F^i(t^i)}{f^i(t^i)} - \frac{\lambda^{0i}}{\delta} \frac{\Lambda^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right] \tag{4}$$

subject to:

$$\begin{aligned}
& 0 \leq \sum_{i \in B} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right] \\
& \quad + \sum_{i \in S} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i + \frac{F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right] \\
0 \quad & \leq \quad \delta - \overline{\lambda} \\
0 \quad & = \quad (\delta - \overline{\lambda}) \{ \sum_{i \in B} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right] \\
& \quad + \sum_{i \in S} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i + \frac{F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right] \}
\end{aligned}$$

[13] Of course the optimal value of δ may vary with the welfare weights.

5.2.3 *Auctions:* Many buyers and one seller (or one buyer and many sellers)

The problem of designing revenue-maximizing auctions when buyers have independent private values was initiated by Vickrey (1961), but not solved until 1981, when three papers were published almost simultaneously by Harris and Raviv (1981), Myerson (1981), and Riley and Samuelson (1981).

Here, we address a more general version of the problem, characterizing all *interim efficient* auctions. The expected revenue maximizing auction[14] arises as a special case, which corresponds in our framework to setting all the buyers' welfare weights to 0, and setting the seller's welfare weights to a positive constant. For that special case, itis already well known that the optimal mechanism can be implemented many simple ways, such as a second price auction with a publicly announced reserve bid, where the reserve bid is a function of the seller's type.

In the general case with type-dependent seller weights, the implementation of optimal mechanisms by auctions can be much more complicated, in particular, secret reserve bids and bid-dependent reserve bids may be optimal. This is true, even if the buyer welfare weights are equal to 0. If buyer welfare weights are positive, the problem is even further complicated. At the opposite extreme, where all the weight is on the buyers' welfare, the problem become equivalent to the general assignment problem, which is analyzed in the next section.

Denote the seller by s, and the buyers by $i = 1, ..., n$. Recall that, for the buyers, $(1 - \frac{\lambda^{0i}}{\delta})M^i(Q^i(\cdot), U^{0i}) = 0$, and for the sellers, $(1 - \frac{\lambda^{0s}}{\delta})M^s(Q^s(\cdot), U^{0s}) = (1 - \frac{\lambda^{0s}}{\delta})\int_{\underline{t}^s}^{\overline{t}^s} Q^s(t^s)dt^s$. Therefore, it follows from Theorem XX that $\exists a^*$ such that (x^*, a^*) is an interim efficient auction if and only if there exist nonegative functions $\lambda^s(t^s)$, $\{\lambda^i(t^i)\}_I$, not all 0, and δ such

[14] Formally, this is only revenue maximization if the seller's type is 0. It would be more precise to call this expected profit maximization, where the seller's type can be viewed as the cost.

that x^* maximizes

$$
\begin{aligned}
& \int_{\underline{t}^s}^{\overline{t}^s} \left(t^s + \frac{F^s(t^s)}{f^s(t^s)} - \frac{\lambda^{0s}}{\delta} \frac{\Lambda^s(t^s)}{f^i(t^s)} \right) Q^s(t^s) dF^s(t^s) \\
& + \sum_{i=1}^{n} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} + \frac{\lambda^{0i}}{\delta} \frac{1 - \Lambda^s(t^s)}{f^i(t^s)} \right) Q^i(t^i) dF^i(t^i) \right]
\end{aligned}
$$

$$
\begin{aligned}
\text{and} \quad : & \\
& 0 \leq \int_{\underline{t}^i}^{\overline{t}^i} \left(t^s + \frac{F^s(t^s)}{f^s(t^s)} \right) Q^s(t^s) dF^s(t^s) \\
& + \sum_{i=1}^{n} \int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \\
0 \quad \leq & \quad \delta - \overline{\lambda} \\
0 \quad = & \quad (\delta - \overline{\lambda}) \{ \int_{\underline{t}^s}^{\overline{t}^s} \left(t^s + \frac{F^s(t^s)}{f^s(t^s)} \right) Q^s(t^s) dF^s(t^s) + \\
& \sum_{i=1}^{n} \int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \}
\end{aligned}
$$

We next show how the familiar revenue maximization problem falls out of our framework.

Revenue maximization For revenue maximization, assume that $\lambda^i(t^i) = 0$ for all i and t^i, and $\lambda^s(t^s) = 1$ for all t^s. This implies that welfare is maximized by maximizing the expected surplus to the seller. It follows immediately from theorem XX that $\exists a^*$ such that

(x^*, a^*) is an interim efficient auction if and only if there exists $\delta \geq 1$ such that x^* maximizes

$$\begin{aligned}
& \int_{\underline{t}^s}^{\overline{t}^s} \left(t^s + (1 - \frac{1}{\delta}) \frac{F^s(t^s)}{f^s(t^s)} \right) Q^s(t^s) dF^s(t^s) \\
& + \sum_{i=1}^{n} \left[\int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right] \\
\text{and} \quad : & \\
& 0 \leq \int_{\underline{t}^i}^{\overline{t}^i} \left(t^s + \frac{F^s(t^s)}{f^s(t^s)} \right) Q^s(t^s) dF^s(t^s) \\
& + \sum_{i=1}^{n} \int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \\
0 \quad \leq \quad & \delta - 1 \\
0 \quad = \quad & (\delta - 1) \{ \int_{\underline{t}^s}^{\overline{t}^s} \left(t^s + \frac{F^s(t^s)}{f^s(t^s)} \right) Q^s(t^s) dF^s(t^s) \\
& + \sum_{i=1}^{n} \int_{\underline{t}^i}^{\overline{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \}
\end{aligned}$$

Assuming individual rationality is not binding on the seller, the inequality constraint is slack, so $\delta = 1$. Assuming we are in the regular case, this gives us the following well-known solution.

Proposition 1 *Pick any buyer* $i^* \in \arg\max_i \left\{ t^i - \frac{1-F^i(t^i)}{f^i(t^i)} \right\}$. *If* $t^{i^*} - \frac{1-F(t^{i^*})}{f(t^{i^*})} \geq t^s$, *then sell to* i^*. *Otherwise do not sell.*

Remark 18 *In the regular case, if all buyers draw valuations from the same distribution, this corresponds to direct mechanism where a sale is made if and only if* $t^i \geq \widetilde{t}$, *for some buyer, where* $\widetilde{t} - \frac{1-F(\widetilde{t})}{f(\widetilde{t})} = t^s$. *Buyer* i *pays a price equal to* $t^i - \frac{1-F(t^i)}{f(t^i)}$ *and other buyers pay* 0. *This allocation can be implemented by many different kinds of auctions; for example, the seller could publicly announce a reserve bid equal to* $\widetilde{t}$ *and then hold a first or second price auction. If buyers' valuations are drawn from different distributions then each buyer* i *would* *have a personalized reserve bid, $\tilde{t}^i$ defined by $\tilde{t}^i - \frac{1-F^i(\tilde{t}^i)}{f^i(\tilde{t}^i)} = t^s$. As Myerson noted, profit maximizing auctions are generally inefficient for two reasons. First, sellers restrict sales by use of the reserve bid, so sometimes the good is not sold even when all buyers value it more than the seller. Second, sellers discriminate between buyers with different value distributions, so even if the good is sold, it may not be purchased by the highest valuation buyer.*

Interim efficient auctions that are not revenue maximizing We next consider the case where the welfare weights are still concentrated on the seller, but the welfare weights are not the same for all seller types, so that $\lambda^i = 0$ for $i = 1, ..., n$ as before, but $\lambda^s(t^s)$ is not constant.[15] This case is more interesting for two reasons. First, $F^s - \Lambda^s \neq 0$, so there will be cross subsidization of seller types. Second it is possible that $\delta > 1$, if there is sufficient cross subsidization that individual rationality is binding on some seller types. This could arise, for example, if some sellers whose valuations are in the support of the buyers' valuations are earning 0 profits).

Without loss of generality, we can normalize $\lambda^s(t^s)$ so that $\lambda^{0s} = 1$. By doing so, for suitably chosen δ the maximand in expression YY becomes:

$$\int_{\underline{t}^s}^{\bar{t}^s} \left(t^s + \frac{F^s(t^s)}{f^s(t^s)} - \frac{1}{\delta} \frac{\Lambda^s(t^s)}{f^i(t^s)} \right) Q^s(t^s) dF^s(t^s)$$
$$+ \sum_{i=1}^{n} \left[\int_{\underline{t}^i}^{\bar{t}^i} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) Q^i(t^i) dF^i(t^i) \right]$$

or

$$\int_T \left[\left(t^s + \frac{F^s(t^s)}{f^s(t^s)} - \frac{1}{\delta} \frac{\Lambda^s(t^s)}{f^i(t^s)} \right) q^s(t) + \sum_{i=1}^{n} \left(t^i - \frac{1 - F^i(t^i)}{f^i(t^i)} \right) q^i(t) \right] dF^s(t^s).$$

Because δ is chosen so that individual rationality is just binding, we must have $\delta = 1$. Hence,

[15] For example, λ^s decreasing corresponds to a seller who is more concerned about earning profits when his valuation is low than when his valuation is high.

it is optimal[16] for each seller type t^s to set bidder-specific reserve bids, each of which is a price,$\widetilde{t}_i$, satisfying

$$\widetilde{t^i} - \frac{1 - F^i(\widetilde{t^i})}{f^i(\widetilde{t^i})} = t^s + \frac{F^s(t^s) - \Lambda^s(t^s)}{f^s(t^s)}.$$

Thus we can see that the *reserve bid principle* continues to hold. That is, the optimal auction corresponds to a direct mechanism in which the seller rejects any bid less than $\widetilde{t^i}$. The standard construction of the a^* indicates that this can be implemented by a second price auction with a reserve bid, where the second price is the maximum of $\widetilde{t^i}$ and the second highest bid. However, there are two important differences. First, the reserve bids must be made secret.[17] Second, the seller must commit to the (secret) reserve bid rule, since the reserve bid does not maximize interim expected profits, except in the special case when $F^s(t^s) = \Lambda^s(t^s)$. For example, if seller welfare weights are increasing then reserve bids will tend to be higher, since $F^s(t^s) > \Lambda^s(t^s)$, and the good is sold less often. If seller welfare weights are decreasing, then reserve bids will tend to be lower.

Finally, suppose the welfare weight on buyers is not zero. Then we are back to the market setting with $J = 1$.

5.2.4 ***Assignment*: J objects, no sellers**

There are two cases to distinguish here, depending on whether the individuals share prior claims to the objects. If they do, then we are in an environment similar to Cramton, Gibbons, and Klemperer (1987), who explore some properties of ex ante efficiency for the $J = 1$ case, when the I individuals share property rights on a single object. Again, this leads to a model where individual rationality constraints are type specific.

[16] We are still assuming the regular case.

[17] With publicly announced reserve bids, the best you can do is to set the interim profit maximizing reserve bid: $\widetilde{t^i} - \frac{1-F^i(\widetilde{t^i})}{f^i(\widetilde{t^i})} = t^s$.

Here we deal with the second case, where there are no prior claims, so the individual rationality constraints are the same as in the standard auction problem ($U^i \geq 0$). Ex ante efficiency has been characterized by Dudek, Kim, Ledyard (1995).

The problem is a special case of the multilateral bargaining environment studied in XX, with $\lambda^i(t^i) = 0$ for all $i \in S$ and for all $t^i \in T^i$, and $t^s = 0$ for all s. Then $C(x) = 0$, $q^i(x) = y^i$, $U^i = t^i y^i - a^i$ and feasibility requires $0 \leq q^i \leq 1$, and $0 \leq \sum_{i=1}^n q^i \leq J$. Normalizing $\lambda^{0i} = 1$ for the buyers, we have $W^i(t^i) = t^i - \frac{1-F^i(t^i)}{f^i(t^i)} + \frac{1}{\delta}\frac{1-\Lambda^i(t^i)}{f^i(t^i)}$. Since the sellers' interim utility is given zero weight in the welfare function, $W^i(t^i) = 0$ for all $i \in S$. Assuming regularity, the interim efficient mechanism picks the J largest i such that:

$$W^i(t^i) = t^i - \frac{1-F^i}{f^i} + \frac{\lambda^{0i}}{\delta}\frac{1-\Lambda^i(t^i)}{f^i(t^i)} \geq 0$$

and sets $q^i_\delta = 1$ for each of these agents, where δ is the smallest value greater than or equal to 1 such that:

$$\sum_{i=1}^n \int \left(t^i - \frac{1-F^i(t^i)}{f^i(t^i)} \right) Q^i_\delta(t^i) dF^i(t^i) \geq 0 \text{ for all } i.$$

If there are fewer than J agents for whom $W^i(t^i) \geq 0$, then some of the goods are allocated to nobody.[18]

If $F^i(t^i) = \overline{F}$ and $\Lambda^i = \overline{\Lambda} = 1$ for all i then in the regular case interim efficient rules (with free disposal) award the items to the subset of the J highest t^i whose values are larger than some critical value, t^o. This t^o is the value of t for which $W^i(t, \lambda, \delta) = 0$ when δ is chosen so that the individual rationality constraint just binds. This can be accomplished in a variety of ways, such as entry fees and reserve bids, depending on the exact details of the environment. The critical value is dependent on $\overline{F}$ and $\overline{\Lambda}$. Let $Q^i(t^i) =$ prob $\{t^i \geq t^J\}$ where t^J is the

[18]This assumes free disposal. If all goods *must* be allocated, then some units may be allocated to agents with negative virtual valuations.

J-highest t^j of all $j \neq i$. Let t^δ solve min $\int_{t^\delta}^{\bar{t}} \left(t - \frac{1-F^i(t^i)}{f^i(t^i)}\right) Q(t)dF(t) \geq 0$. Notice that as we change δ the ranking of the t^i stays exactly the same as the ranking of the $W^i(t^i, \lambda, \delta)$, which directly implies that the items are always awarded to the highest $t^i \geq t^\delta$. So δ only affects whether $W^i(t^\delta, \lambda, \delta) \geq 0$. Finding t^δ above is equivalent to minimizing δ. At this value of δ, $W^i(t^\delta, \lambda, \delta) = 0$. This result is also independent of $\Lambda(t)$, the interim weights in the symmetric case.

When F is uniform on $[0, 1]$ then it is possible to show that individual rationality constraints are never binding, so $\delta = 1$. In fact, *any* feasible incentive compatible mechanism is individually rational in the uniform case. To see this, notice first that incentive compatibility implies Q^i is increasing for all i. Therefore, to prove that individual rationality is not binding, it is sufficient to show that $\int_0^1 (2t^i - 1) Q^i(t^i)dt^i \geq 0$ for all non-decreasing functions Q^i. Integrating $\int_0^1 (2t^i - 1) Q^i(t^i)dt^i$ by parts gives

$$\begin{aligned}
\int_0^1 \left(2t^i - 1\right) Q^i(t^i)dt^i &= \int_0^1 Q^i(t^i)dt^i - \int_0^1 2\left[\int_0^{t^i} Q^i(s^i)ds^i\right] dt^i \\
&\geq \int_0^1 Q^i(t^i)dt^i - \int_0^1 2t^i Q^i(t^i)dt^i \\
&= -\int_0^1 \left(2t^i - 1\right) Q^i(t^i)dt^i \\
\text{Therefore} \int_0^1 \left(2t^i - 1\right) Q^i(t^i)dt^i &\geq 0
\end{aligned}$$

The second step follows because $\int_0^{t^i} Q^i(s^i)ds^i \leq t^i Q^i(t^i)$ for all nondecreasing Q^i. Therefore, it follows that in the uniform case, $\delta = 1$, regardless of the welfare weights. So, the virtual valuations reduce to:

$$W^i(t^i, \lambda^i) = 2t^i - 1 + \int_{t^i}^1 \lambda^i(t^i)dt^i$$

This implies immediately that the ex ante efficient solution coincides with the ex post efficient solution in the uniform case. The same is true for any interim efficient mechanism, provided $\lambda^i(t^i) \leq 2$ (i.e. the regular case), since

$$\begin{aligned} W^i(0, \lambda^i) &= 2t^i - 1 + \int_0^1 \lambda^i(t^i)dt^i \\ &= 0 - 1 + \lambda^{0i} \\ &= 0 \end{aligned}$$

.

5.2.5 Complementarities and Single-minded buyers

The optimal mechanism ranks all i by W^i and awards an item to the M highest: buyers with high t^i get an item, sellers with high t^i keep their item. We turn now to an application of these methods to the problem of allocating M objects to N people who have preferences for bundles of discrete private goods. We will illustrate this with the special case of *single-minded buyers*, which is an extreme case of complementarity.[19] Specifically, each person will be identified by a unique subset $S^i \subseteq M$ of the M objects which they want and a utility function such that $U^i = t^i q^i(z) - a^i$ where $z \subseteq M$ and $q^i(M) = 1$ if $S^i \subseteq M$ and $q^i(M) = 0$ otherwise.[20] This is another special case of our structure in which interim efficient mechanisms pick q to

$$maximize \sum_{i=1}^{n} W^i(t^i)q^i$$

[19] Levin (1997) investigates a similar model for the case of two goods.

[20] Ideally we would like to analyze the case in which $u^i = \sum_{s \subseteq M} q_s^i t_s^i$ where $q_s^i = 1$ if i gets object s. But that involves multi-dimensional types.

$$
\begin{aligned}
\text{S.T.} \qquad\qquad q^i &\in [0,1] \\
q^i &= 1 \text{ if } C_k^i = 1 \forall k \in s^i \\
&\quad\ 0 \text{ otherwise} \\
\text{and} \quad \textstyle\sum_{i=1}^n C_k^i &= 1 \ \forall k
\end{aligned}
$$

and then minimize α. A full analysis of this problem remains to be done but a simple example offers some insight into the nature of efficient mechanisms in these environments. Suppose $N = 3$, $S^1 = \{a\}$, $S^2 = \{b\}$, and $S^3 = \{a, b\}$. Further assume $t^i \sim$ uniform $[0, 1$ for each i. The ex ante efficient mechanism awards the pair of items to 3 if $W^3(t, \alpha) = t^3 - \alpha\frac{1-F}{f} \geq t^2 - \alpha\frac{1-F}{f} + t^3 - \alpha\frac{1-F}{f}$. It awards them to 1 and 2 otherwise. In this case $\frac{1-F}{f} = 1 - t$ so 3 wins to iff $t^3 - \alpha(1-t^3) \geq t^1 - \alpha(1-t^i) - \alpha(1-t^2)$ or iff $t^3 \geq t^1 + t^2 - \frac{\alpha}{1+\alpha}$. So if individual rationality is binding the items can be awarded to 3 even though the first best allocation would award them to 1 and 2. This happens when $t^3 + \frac{\alpha}{1+\alpha} > t^1 + t^2 > t^3$. If one wants to maximize expected revenue, simply set $\alpha = 1$ so now 3 wins whenever $t^3 + \frac{1}{2} \geq t^1 + t^2$. The optimal ex ante auction is *not* first best efficient even if types are one dimensional and utility is linear in type.

If there is an option not to sell then each i should be excluded when $t^1 - (1 - t^i) = 2t^i - 1 < 0$ or $t^i < 1/2$. So let $t^i = \max\{2t^i - 1, 0\}$. Then, if $t^i \geq 1/2$ for all i, 3 wins iff $W^3 \geq W^1 + W^2$. 1 and 2 win otherwise. However, the goods will not be awarded to an agent for whom $2t^i - 1 < 0$. Hence the solution is summarized in the following table:

	$t^3 < \frac{1}{2}$	$t^3 > \frac{1}{2}$
$t^1 < \frac{1}{2}, t^2 < \frac{1}{2}$	No one	3 wins
$t^1 < \frac{1}{2}, t^2 > \frac{1}{2}$	only 2 wins	3 wins if $t^3 \geq t^2$ 2 wins if $t^3 \geq t^3$
$t^1 > 1/2, t^2 < \frac{1}{2}$	only 1 wins	3 wins if $t^3 \geq t^1$ 1 wins if $t^1 \geq t^3$
$t^1 > \frac{1}{2}, t^2 > 1/2$	1 and 2 win	3 wins if $t^3 + \frac{1}{2} \geq t^1 + t^2$ $1 + 2$ win otherwise

Buyers 1 and 2 each win with probability equal to 27/64. Buyer 3 wins with probability equal to 17/64. This compares to $\frac{3}{4}$ and $\frac{1}{4}$, respectively, under a first-best allocation, so agent 3 wins "too often", and the other two agents are excluded too often, relative to the first best.

6 Conclusions

This paper presented a general framework to study the theoretical properties of interim efficient mechanisms in independent linear environments. Interim efficient allocation rules are fully characterized for these environments. For regular environments, the solution is often obtainable by applying classical welfare analysis, substituting easily computable *virtual utilities* for the agents' actual utilities. We illustrated this approach with a series of applications, some of which has been studied elsewhere in the literature, including both public goods and private goods applications. Other applications can also be analyzed in a similar way, including the problem of optimal cartel agreements (Cramton and Palfrey 1990), optimal reallocation of a jointly owned asset (Cramton, Gibbons, and Klemperer 1987), optimal regulatory mechanisms (Baron and Myerson 1982), transfer pricing in organizations, and so forth.

Several directions for future research seem promising. First, the incoporation of common or affiliated values can be done, at least for some specifications. For example, Myerson's (1981) revision effects can be incorporated with only minor adjustments to the virtual valuations. A second issue, correlated types, inovolve some special features that we do not consider here, namely using complicated side-payments schemes that exploit the correlation in order to relax incentive constraints. These are used elsewhere, for example Cremer and McLean (1988) and indeed can often relax incentive constraints fully, so that first best is acheivable. However, due to the complicated nature of the sidepayments, these mechanisms may be impractical in most situations and also fail if there are limited liablility constraints or if collusion is possible (Laffont and Martimort 2000). Third, there are interesting open questions about the asymptotic properties of interim efficient allocations. Fourth, the applications studied here only considered the regular case, and the exact details of efficient mechanisms for these applications in the irregular case is not fully solved. While it is known that optimal mechanisms will have flat regions in the irregular case, a deeper understanding may be required in order to answer general questions about the asymptotic properties of the set of interim efficient allocations.

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